

# Are electroweak corrections at 1 TeV under control at the 1 % level?

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Future lepton colliders will provide a powerful tool for making precision experiments at energies that will range typically between 500 GeV and 2 TeV. At such high energies, one loop electroweak corrections are bigger than one could naïvely expect *a priori*. Thus, the calculation of higher order electroweak effects (and possibly their resummation) might be needed.

## 1 Introduction

High precision experiments at LEP have been able to prove the quantum structure of the electroweak theory at the *per mille* level<sup>1</sup>. These experiments have tested the Standard Model (SM) electroweak sector at energies close to the Z mass  $M_Z \approx 91$  GeV. The typical magnitude of SM electroweak corrections is dictated at LEP by the perturbative series expansion parameter  $\frac{\alpha(M_Z)}{4\sin^2\theta_w\pi} \approx 2.7 \times 10^{-3}$ , where  $\alpha(M_Z)$  is the QED effective coupling constant at the energy  $M_Z$  and  $\sin\theta_w$  is the Weinberg angle. Since the experimental accuracy is of comparable magnitude, the well known one-loop electroweak corrections<sup>2</sup> are sufficient in general to allow for a comparison between theory and experiment. As an exception, some leading two loop electroweak corrections<sup>3</sup> growing with the top mass  $m_t$  turn out to be also relevant at LEP.

While experiments at LEP have tested the electroweak theory at its characteristic energy of about 100 GeV, future experiments will go much beyond this mass scale. This holds in particular for the generation of linear colliders<sup>4</sup>. These colliders will feature high luminosities, allowing for precision experiments at energies ranging from 500 GeV to 2 TeV. The calculation of electroweak corrections at such high energies with a precision comparable to the experimental accuracy is then an important issue. Recent results<sup>5</sup> seem to indicate that yet uncalculated higher order electroweak effects and/or their possible resummation are indeed important for future linear colliders.

## 2 IR divergences and double logs

What is the order of magnitude of electroweak corrections that one expects at a typical energy of, say, 1 TeV? Let us assume that we determine the SM parameters with high precision through a series of LEP experiments at the Z

mass. Then we expect that perturbative corrections for an observable measured at a different c.m. energy  $\sqrt{s}$  are enhanced by large logarithms of ultraviolet (UV) origin of the form  $\frac{\alpha(M_Z)}{4\sin^2\theta_w\pi} \log \frac{s}{M_Z^2} \approx 1.3 \times 10^{-2}$  for  $\sqrt{s} = 1$  TeV. Since the one loop effects are of the order of 1 %, we expect higher order effects to be of the order of 0.1 %. Moreover, the large logarithms can be resummed at all orders through renormalization group equations (RGEs). Then, if we fix the expected experimental accuracy to be at the 1 % level at NLCs (which is probably a conservative assumption since the accuracy is expected to be better than this<sup>4)</sup>, there is no need to worry at all: electroweak corrections are under control, i.e. they are theoretically known through one-loop results with an accuracy which is better than the experimental one.

However, this is not the end of the story. As has already been noticed<sup>6)</sup>, electroweak corrections also contain terms growing with the energy  $\sqrt{s}$  like the *square* of a log, i.e. proportional to  $\log^2 \frac{s}{M_{Z,W}^2}$ . This can be understood as follows: when the energy is much bigger than the mass of all the particles running in the loops, which means  $\sqrt{s} \gg M_W, M_Z$  if we don't consider processes in which the top quark plays a role, the W and Z mass act as an effective cutoff for infrared (IR) divergences. Infrared divergences arise in perturbative calculations from regions of integration over the loop momentum  $k$  where  $k$  is small compared to the typical scales of the process. This is a well known fact in QED for instance<sup>7)</sup> where the problem of an unphysical divergence is solved by giving the photon a fictitious mass which acts as a cutoff for the IR divergent integral. When real (bremsstrahlung) and virtual contributions are summed, the dependence on this mass cancels and the final result is finite<sup>7)</sup>. The (double) logarithms coming from these contributions are large and, growing with the scale, can spoil perturbation theory and need to be resummed. They are usually called Sudakov double logarithms<sup>8)</sup>. In the case of electroweak corrections, similar logarithms arise when the typical scale of the process considered is much larger than the mass of the particles running in the loops, typically the  $W(Z)$  mass<sup>6,9,10)</sup>. The expansion parameter results then  $\frac{\alpha}{4\sin^2\theta_w\pi} \log^2 \frac{s}{M_W^2}$ , which is already 7 % for energies  $\sqrt{s}$  of the order of 1 TeV. In the case of corrections coming from loops with  $W(Z)$ s, there is no equivalent of “bremsstrahlung” like in QED or QCD: the  $W(Z)$ , unlike the photon, has a definite nonzero mass and is experimentally detected like a separate particle. In this way the full dependence on the  $W(Z)$  mass is retained in the corrections. Let us consider IR divergences coming from vertex corrections for instance (also box diagrams are present in two fermion production, but I do not discuss them here). For simplicity, I consider a “SM-like case” in which a “W boson” having mass  $M$  and coupling with fermions like the

photon is exchanged. In the limit of massless fermions considered here, there is no coupling to the Higgs sector. Moreover, by power counting arguments, it is easy to see that the vertex correction where the trilinear gauge boson coupling appears is not IR divergent. The only potentially IR divergent diagram is then the one of fig. 1, where a gauge boson is exchanged in the t-channel. It is convenient to choose the momentum of integration  $k$  to be the one of the exchanged particle, the boson in this case. Then, by simple power counting arguments it is easy to see that the IR divergence can only be produced by regions of integration where  $k \approx 0$ . The only potentially IR divergent integral is then the scalar integral, usually called  $C_0$  in the literature<sup>11</sup>. Any other integral with  $k_\mu, k_\mu k_\nu$  in the numerator cannot, again by power counting, be IR divergent. To understand the origin of the divergences, let us consider the diagram of fig.1 with all the masses set to zero. For  $k \approx 0$  the leading term of the vertex amplitude is given by:

$$\mathcal{V} \approx -\frac{\alpha}{4\pi} \mathcal{V}_0 \int \frac{d^4 k}{i\pi^2} \frac{(p_1 p_2)}{k^2 (k p_1)(k p_2)} \approx -\frac{\alpha}{2\pi} \mathcal{V}_0 \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y} \quad (1)$$

where  $\mathcal{V}_0$  is the tree level vertex. We can see here the two logarithmic divergences that arise from the integration over the  $x, y$  Feynmann parameters. As is well known<sup>7</sup>, one of them is of collinear origin and the other one is a proper IR divergence. When we take some of the external squared momenta and/or masses different from zero, they serve as cutoffs for the divergences. The bottom line is that the Feynman diagram of fig. 1 produces a term proportional to  $\alpha \log^2 \frac{s}{M^2}$ , where  $M$  is the exchanged boson mass. It is easy to see that the dependence on the IR logs simply factorizes for the cross section ( $\sigma_B$  is the tree level cross section):

$$\sigma \propto \frac{1}{s} \int_0^s \frac{dt}{s} |\mathcal{M}_0|^2 [1 - 2 \frac{\alpha}{4\pi} \log^2 \frac{s}{M^2}] = \sigma_B [1 - 2 \frac{\alpha}{4\pi} \log^2 \frac{s}{M^2}]$$

### 3 Asymptotic behavior of two fermion processes

In<sup>5</sup> and<sup>12</sup>, the production of two massless fermions in an high energy lepton collider has been considered. In<sup>5</sup> the coefficients of the leading terms, growing with the energy like the square of a log as we have seen, have been calculated. In<sup>12</sup> also the coefficients of the subleading terms, growing like a single log and of collinear origin, have been calculated for various observables. In the following, I call “Sudakov-type” logs the single and double logs of IR and collinear origin, to distinguish them from the logs of UV origin. As an example, let me consider

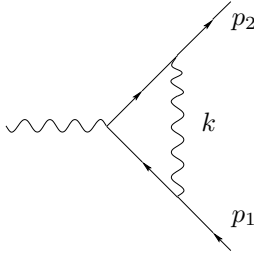


Figure 1: Vertex diagram generating a  $\log^2 \frac{s}{M^2}$ .  $p_1$  and  $p_2$  are ingoing.

the total cross section for the process  $e^+e^- \rightarrow \mu^+\mu^-$ . From<sup>12</sup>, we get:

$$\sigma_\mu \stackrel{\sqrt{s} \gg M}{\approx} \sigma_B \left\{ 1 + \frac{\alpha}{4\pi \sin^2 \theta_w} [0.6 L_{UV} + 9.4 L_{IR} - 1.4 L_{IR}^2] \right\} \quad (2)$$

Here,  $L_{UV}$  and  $L_{IR}$  are numerically the same:  $L_{UV} = L_{IR} = \log \frac{s}{M^2}$  and  $M$  is the weak scale  $M \approx M_W \approx M_Z \approx 91$  GeV.  $\sigma_B$  is the Born cross section, precisely defined in<sup>12</sup>. This formula is expected to describe the full one loop calculation better and better as the energy grows, since the subleading terms that have not been extracted become less and less important with respect to the leading logarithmic ones. Indeed, formula 2 well approximates the exact result coming from numerical programs<sup>13</sup> for energies around 1 TeV (see<sup>12</sup>). However, for energies well above 1 TeV where the agreement is supposed to be even better, no numerical computation is available at the moment. The graph corresponding to eq. 2 is drawn in fig. 2, where the relative deviation for the total cross section  $\frac{\Delta\sigma}{\sigma} \equiv \frac{\sigma - \sigma_B}{\sigma_B}$  is drawn as a function of the c.m. energy, and the various contributions are also separately plotted.

One evident feature of eq. 2 is that, while the coefficients of the single UV log and the double Sudakov log are of order 1, as one could expect *a priori*, the coefficient of the single Sudakov-type single log is of order 10. This has two immediate consequences:

- The contribution of the single log of UV origin is almost negligible with respect to the Sudakov logs ones. Thus, a naïve expectation of an asymptotic behavior dictated by the UV structure of the theory turns out to be wrong.
- Since the sign of the double and single Sudakov logs are opposite, there are big cancellations and the correction to  $\sigma_\mu$  crosses a zero at an energy of about 2 TeV

These features are most easily seen by looking at fig. 2, where the net result is seen to result from cancellations of big contributions of Sudakov single and double logs of opposite signs, while the RGE driven logs are almost negligible.

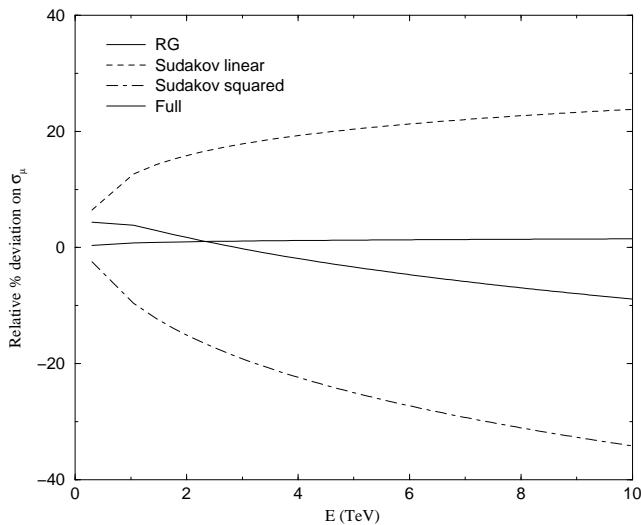


Figure 2: Relative deviation  $\frac{\Delta\sigma}{\sigma}$  for the total cross section of  $e^+e^- \rightarrow \mu^+\mu^-$

Where does all this leave us with the question posed with the title? In the end, one could think that when doing perturbative calculations, big cancellations can *always* be present (between different graphs contributing to the same amplitude for instance). Then, since here the net effect is only a few percent in the considered energy range as one can see from fig. 2, there is no need to worry about higher order effects. However, the situation here is different in my opinion. Here we have terms that are separately gauge invariant and have different energy behavior. The fact that their contribution almost exactly cancels at an energy of about 2 TeV which is close to the energy of interest, is to be taken as accidental. Let us take another point of view: the relative effect of double logs is, from eq. 2,  $1.4 \frac{\alpha}{4\pi \sin^2 \theta_w} \log^2 \frac{s}{M^2} \approx 0.1$  at 1 TeV. A two loop calculation will produce a term growing like the fourth power of a log, of the order  $(0.1)^2 = 1\%$ . Until an higher order calculation will be done, one cannot say that electroweak corrections at 1 TeV are under control at the 1 % level.

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